Definition 1:

A **quadratic equation** in one variable $x$ is a second degree polynomial defined as,

$$ax^2 + bx + c = 0$$

where $a$, $b$ and $c$ are real numbers, and $a$ is strictly not equal to zero.
1. FACTORING

2. COMPLETING THE SQUARE

3. QUADRATIC FORMULA

Three Ways of Solving a Quadratic Equation
Objectives: Solving by Factoring

- Apply the concept of factoring and PZP in solving quadratic equations.
- Independently answer the seatwork related to the topic.
Essential Question:

HOW DO THE FACTORS OF A QUADRATIC POLYNOMIAL RESULT TO THE ROOTS OF A QUADRATIC EQUATION?
Interpret the factored equation:

\[(x - 3)(x + 2) = 0\]
For which value of $x$ will this be true?

$$(x - 3)(x + 2) = 0$$
How do you show this graphically?

$$(x - 3)(x + 2) = 0$$
Think about the questions and discuss with your seatmate.

\[(x - 3)(x + 2) = 0\]
Interpret the factored equation:

$$(x - 3)(x + 2) = 0$$

The equation is telling us that the product of two numbers is equal to zero.
For which value of $x$ will this be true?

$$(x - 3)(x + 2) = 0$$

The equation is true whenever $x = 3$ or $x = -2$. Hence, $\{-2, 3\}$ are called the **roots** of the quadratic equation.
How do you show this graphically?

\[(x - 3)(x + 2) = 0\]
For any real numbers $a$ and $b$, if $ab=0$, then $a=0$ or $b=0$
How do we apply PZP in this equation???

\[(x - 3)(x + 2) = 0\]

\[x^2 - x - 6 = 0\]
Steps in Solving Quadratic Equations using Factoring:

\[ x^2 - x = 20 \]

- Write the quadratic equation in its standard form.
- Factor out the quadratic equation.
- By PZP, equate each factor to zero.
- Solve the linear equation.
- Check whether the values satisfy the given equation.
### Quadratic Equation: Review of Factoring

<table>
<thead>
<tr>
<th>Quadratic Equation</th>
<th>Sum</th>
<th>Product</th>
<th>Two Numbers</th>
<th>Factored Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^2 + 12x + 35 = 0$</td>
<td>12</td>
<td>35</td>
<td>7, 5</td>
<td>$(x + 7)(x + 5) = 0$</td>
</tr>
<tr>
<td>$x^2 - 15x + 50 = 0$</td>
<td>-15</td>
<td>50</td>
<td>-5, -10</td>
<td>$(x - 10)(x - 5) = 0$</td>
</tr>
<tr>
<td>$x^2 + 2x - 8 = 0$</td>
<td>2</td>
<td>-8</td>
<td>-2, 4</td>
<td>$(x + 4)(x - 2) = 0$</td>
</tr>
<tr>
<td>$x^2 + 3.5x + 2.5 = 0$</td>
<td>3.5</td>
<td>2.5</td>
<td>2.5, 1</td>
<td>$(x + 2.5)(x + 1) = 0$</td>
</tr>
<tr>
<td>$8x^2 + 6x + 1 = 0$</td>
<td>$\frac{3}{4}$</td>
<td>$\frac{1}{8}$</td>
<td>$\frac{1}{2}, \frac{1}{4}$</td>
<td>$(2x + 1)(4x + 1) = 0$</td>
</tr>
</tbody>
</table>

**How do you factor out a quadratic trinomial?**
<table>
<thead>
<tr>
<th>Quadratic Equation</th>
<th>Factored Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^2 + 12x = 0$</td>
<td>$x(x - 12) = 0$</td>
</tr>
<tr>
<td>$x^2 = 2x$</td>
<td>$x(x - 2) = 0$</td>
</tr>
<tr>
<td>$x^2 = 9$</td>
<td>$(x + 3)(x - 3) = 0$</td>
</tr>
<tr>
<td>$x^2 - 169 = 0$</td>
<td>$(x - 13)(x + 13) = 0$</td>
</tr>
<tr>
<td>$4x^2 - 9 = 0$</td>
<td>$(2x + 3)(2x - 3) = 0$</td>
</tr>
</tbody>
</table>

Other forms of factoring:

Can you remember factoring out the GCF? Difference of two squares?
Essential Question:

How do the factors of a quadratic polynomial result to the roots of a quadratic equation?
Three Ways of Solving a Quadratic Equation

1. FACTORING

2. COMPLETING THE SQUARE

3. QUADRATIC FORMULA
Skills you have to possess:

- **IMAGINARY NUMBERS**
  - The imaginary number $i$ is defined as,

  $$i^2 = -1$$

  - Then, the principal root of $-1$ is $i$, or

  $$i = \sqrt{-1}$$
Skills you have to possess:

- IMAGINARY NUMBERS

Simplify the following:

\[
\sqrt{-36} = 6i \\
\sqrt{-8} = 2i \sqrt{2} \\
\sqrt{-27} = 3i \sqrt{3}
\]
Square Root Property

Given: $a$ and $b$ are real numbers and $b > 0$.

$$a^2 = b$$

$$\sqrt{a^2} = \sqrt{b}$$

$$a = +\sqrt{b} \quad | \quad a = -\sqrt{b}$$
Example 1

Skills you have to possess:

Square Root Property

\[ x^2 = 16 \]
\[ \sqrt{x^2} = \sqrt{16} \]

\[ x = +4 \quad \text{or} \quad x = -4 \]
Example 2

Skills you have to possess:

Square Root Property

\[ 5x^2 + 4 = 24 \]

\[ 5x^2 = 20 \]

\[ x^2 = 4 \]

\[ x = +2 \quad \text{or} \quad x = -2 \]
Example 3

Skills you have to possess:

Square Root Property

\[(x + 1)^2 = 35\]

\[\sqrt{(x + 1)^2} = \sqrt{35}\]

\[x + 1 = \pm \sqrt{35}\]

\[x = -1 \pm \sqrt{35}\]
Completing the Square

Recall that:

**PERFECT SQUARE TRINOMIAL:**

\[ x^2 + 2xy + y^2 = (x + y)^2 \]
\[ x^2 - 2xy + y^2 = (x - y)^2 \]
Completing the Square
Recall that to complete a square of a single variable,

>> make the numerical coefficient of the first term equal to 1

>> divide second term by two, then square the quotient to get the third term
Example 1

Skills you have to possess:

Completing the square:

\[ 3x^2 + 18x + \_\_\_\_ \]

\[ \Rightarrow \frac{3x^2 + 18x}{3} + \_\_\_\_ \]

\[ \Rightarrow 3\left(x^2 + 6x + \_\_\_\_\right) \]

\[ \Rightarrow 3\left(x^2 + 6x + \left(\frac{6}{2}\right)^2\right) \]

\[ \Rightarrow 3\left(x^2 + 6x + 9\right) = 3(x + 3)^2 \]

Factor out three.

The last term is the square of half the second term.
Example 2

Skills you have to possess:

Completing the square:

\[ 5x^2 - 5x + \_ \_ \_ \_ \_ \_ \_ \]

\[ \Rightarrow \frac{5x^2 - 5x}{5} + \_ \_ \_ \_ \_ \_ \_ \]

\[ \Rightarrow 5 \left( x^2 - x + \_ \_ \_ \_ \_ \_ \_ \right) \]

\[ \Rightarrow 5 \left[ x^2 - x + \left( \frac{1}{2} \right)^2 \right] \]

\[ \Rightarrow 5 \left( x^2 - x + \frac{1}{4} \right) = 5 \left( x - \frac{1}{2} \right)^2 \]
Example 3

Skills you have to possess:

Completing the square:

\[ ax^2 - bx + \_
\]

\[ \Rightarrow \frac{ax^2 - bx + \_}{\_}
\]

\[ \Rightarrow a \left( x^2 - \frac{b}{a} x + \_ \right)
\]

\[ \Rightarrow a \left[ x^2 - \frac{b}{a} x + \left( \frac{b}{2a} \right)^2 \right]
\]

\[ \Rightarrow a \left( x^2 - \frac{b}{a} x + \frac{b^2}{4a^2} \right) = a \left( x - \frac{b}{2a} \right)^2 \]
Steps in Solving Quadratic Equations using Completing the Square:

- Transpose the constant part on the other side.
- Complete the square of the side with the variable.
  - Whatever you add on one side, you also add it on the other side.
- Write the PST as a binomial.
- Take the square root of the both sides of the equation.
- Solve for the two linear equations.

\[ x^2 + 5x = -7 \]
Example 2

Solve for the roots of $3x^2 - 2x - 6 = 0$

$\rightarrow \frac{3x^2 - 2x = 6}{3}$

$\rightarrow x^2 - \frac{2}{3}x + \frac{1}{3} = 2 + \frac{1}{3}$

$\rightarrow x^2 - \frac{2}{3}x + \left(\frac{1}{3}\right)^2 = 2 + \left(\frac{1}{3}\right)^2$

$\rightarrow \left(x - \frac{1}{3}\right)^2 = \frac{19}{9}$

$\rightarrow \sqrt{\left(x - \frac{1}{3}\right)^2} = \frac{\sqrt{19}}{3}$

$\rightarrow x - \frac{1}{3} = \pm \frac{\sqrt{19}}{3}$

$\rightarrow x = \frac{1}{3} \pm \frac{\sqrt{19}}{3}$

$\rightarrow x = \frac{1 + \sqrt{19}}{3}$
Seatwork:

\[ x^2 + 6x + 41 = 0 \]
\[ 2x^2 - 3x - 11 = 0 \]

\[ x = -3 \pm 4i\sqrt{2} \]
\[ x = \frac{3 \pm \sqrt{97}}{4} \]

HW:

Download the problem set from home notes.
Essential Question:

How do you solve for the roots if the quadratic equation is not factorable?
Answers to HW:

1.) \( \left( x - \frac{1}{3} \right)^2 \)

2.) \( (x - \sqrt{2})^2 \)

3.) \( (2y - 3)(y - 7) \)

4.) \( (9b + 9)(b - 5) \)

5.) \( (4e - f - 1)(2e + f + 1) \)

6.) \( (8x + 5 + 5\sqrt{2})(8x + 5 - 5\sqrt{2}) \)
Answers to HW:

1.) $\frac{1}{4}$

2.) $12y$

3.) $10a$

4.) $\frac{9}{4}$
Answers to HW:

1.) 16

2.) \( \frac{1}{16} \)

3.) \( \frac{1}{25} \)

4.) \( x^2 \)

5.) \( \frac{1}{100} \)

6.) 1

7.) \( \frac{1}{9} \)

8.) \( \frac{49}{16} \)

9.) \( 2h \)
Answers to HW:

1.) \( x = \frac{1 \pm \sqrt{33}}{16} \)

2.) \( x = -1 \pm i \)

3.) \( x = 3 \pm 3i \)

4.) \( x = -3 \pm 2\sqrt{3} \)

5.) \( x = -\frac{9}{2}; x = 1 \)
1. FACTORING

2. COMPLETING THE SQUARE

3. QUADRATIC FORMULA

Three Ways of Solving a Quadratic Equation
Objectives: Solving by Completing the Square

- Apply the concept of completing the square in deriving the quadratic formula.

- Explain the roots of a quadratic equation given the value of the discriminant.
Which is a more general way to solve a quadratic equation: completing the square or quadratic formula?

What is the difference between learning the derivation and memorizing the formula?
Pair Activity 1:

Given this **quadratic equation** in one variable $x$,

$$ax^2 + bx + c = 0$$

where $a$, $b$ and $c$ are real numbers, and $a$ is strictly not equal to zero.

Your task is to solve for the values of $x$ using completing the square. Show your detailed and complete solution in a ½ IPP. Each pair is given 5 minutes to complete the activity.
We are going to solve for \( x \).

\[ a x^2 + b x + c = 0 \]
Theorem: The Quadratic Formula

If $a$ is not equal to zero, the solutions of the equation

$$ax^2 + bx + c = 0$$

are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
Examples:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ x^2 + 3x = 4 \]
\[ x^2 + 3x - 4 = 0 \]
\[ a = 1 \]
\[ b = 3 \]
\[ c = -4 \]
\[ x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-4)}}{2(1)} \]
\[ x = \frac{-3 \pm \sqrt{25}}{2(1)} \]
\[ x = \frac{-3 \pm 5}{2} \]
\[ x = -4; \quad x = 1 \]

\[ 2x^2 = 9x - 5 \]
\[ 2x^2 - 9x + 5 = 0 \]
\[ a = 2 \]
\[ b = -9 \]
\[ c = 5 \]
\[ x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(5)}}{2(2)} \]
\[ x = \frac{9 \pm \sqrt{41}}{4} \]

\[ x^2 - x = -1 \]
\[ x^2 - x + 1 = 0 \]
\[ a = 1 \]
\[ b = -1 \]
\[ c = 1 \]
\[ x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)} \]
\[ x = \frac{1 \pm \sqrt{-3}}{2} \]
\[ x = \frac{1 \pm i\sqrt{3}}{2} \]
Examples:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[
\begin{align*}
\text{x}^2 + 3\text{x} &= 4 \\
\text{x}^2 + 3\text{x} - 4 &= 0 \\
\text{x} &= \frac{-3 \pm \sqrt{25}}{2(1)} \\
\text{x} &= \frac{-3 \pm 5}{2} \\
\text{x} &= -4; \text{x} = 1
\end{align*}
\]

\[
\begin{align*}
2\text{x}^2 &= 9\text{x} - 5 \\
2\text{x}^2 - 9\text{x} + 5 &= 0 \\
\text{x} &= \frac{-9 \pm \sqrt{41}}{2} \\
\end{align*}
\]

\[
\begin{align*}
\text{x}^2 - \text{x} + 1 &= 0 \\
\text{x}^2 - \text{x} + 1 &= 0 \\
\text{x} &= \frac{-1 \pm \sqrt{-3}}{2} \\
\text{x} &= \frac{-1 \pm i\sqrt{3}}{2}
\end{align*}
\]
Characters of the Roots: \[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

The number represented by \( b^2 - 4ac \) is called the discriminant of the quadratic equation.

Case 1: \( b^2 - 4ac = 0 \)
The roots are real and equal. (DOUBLE ROOT)

Case 2: \( b^2 - 4ac > 0 \)
The roots are real and unequal. (RATIONAL OR IRRATIONAL)

Case 3: \( b^2 - 4ac < 0 \)
The roots are imaginary and unequal. (COMPLEX CONJUGATES)
In pairs, copy and answer the following.

1. Kyle increased the area of his garden by 165 sq. ft. The rectangular garden was originally 16 ft by 12 ft, and he increased the length and the width by the same amount. Find the dimensions of the garden now. Illustrate the scenario.

2. The product of two consecutive multiples of three is 180. What are the two numbers?

Seatwork: (1/2 IPP)
Which is a more general way to solve a quadratic equation: completing the square or quadratic formula?

What is the difference between learning the derivation and memorizing the formula?
HW: Answer the problem set.

Prepare for the quiz.